1. FRANCIS L. MIKSA, *Table of quadratic partitions* $x^2 + y^2 = N$, RMT **83**, *MTAC*, v. 9, 1955, p. 198.
2. JOHN LEECH, "Some solutions of Diophantine equations," *Proc. Cambridge Philos. Soc.*, v. 53, 1957, p. 778–780.

39[F].—DAVID C. MAPES, *Fast Method for Computing the Number of Primes less than a Given Limit*, Lawrence Radiation Laboratory Report UCRL-6920, May 1962, Livermore, California. Table of 20 pages deposited in UMT File.

This report is the original writeup of [1]. The table in [1] gives $\pi(x)$, $Li(x)$, $R(x)$, $L(x) - \pi(x)$ and $R(x) - \pi(x)$ for $x = 10^7(10^7)10^9$, where $\pi(x)$ is the number of primes $\leqq x$, and $Li(x)$ and $R(x)$ are Chebyshev's and Riemann's approximation formulas. The table here gives the same quantities for $x = 10^6(10^6)10^9$. It thus has greater "continuity," but not enough to trace the course of $\pi(x)$ unequivocally.

For example, Rosser and Schoenfeld [2] have recently proved that $\pi(x) < Li(x)$ for $x \leqq 10^8$. While it is highly probable that this inequality continues to $x = 10^9$, the gaps here, of $\Delta x = 10^6$, would appear to preclude a *rigorous* proof at this time. Study of the table, however, shows no value of x for which $\pi(x)$ approaches $Li(x)$ sufficiently close to arouse much suspicion. The relevant function is

$$PI(x) = \frac{Li(x) - \pi(x)}{\sqrt{x}} \log x,$$

and for $313 \leqq x \leqq 10^8$, Appel and Rosser [3] showed a minimum value of $PI(x)$, equal to 0.526, at $x = 30{,}909{,}673$. Here (and also in [1]) one finds values of 0.615 and 0.543 at $x = 110 \cdot 10^6$ and $180 \cdot 10^6$, respectively. It is thus likely that a value of $PI(x)$ less than 0.526 can be found in the neighborhood of these x (especially the second), but it is unlikely that $PI(x)$ becomes negative there. The relevant theory [4] is made difficult by incomplete knowledge of the zeta function. In the second half of the table, $x > 500 \cdot 10^6$, no close approaches at all are noted, and $Li(x) - \pi(x)$ exceeds 1000 there, except for $x = 501 \cdot 10^6$, $604 \cdot 10^6$, and $605 \cdot 10^6$.

The low values of $PI(x)$ are always associated with the condition $\pi(x) > R(x)$. The largest value of $R(x) - \pi(x)$ shown here is $+914$, for $x = 905 \cdot 10^6$.

D. S.

1. DAVID C. MAPES, "Fast method for computing the number of primes less than a given limit, "*Math. Comp.*, v. 17, 1963, p. 179–185.
2. J. BARKLEY ROSSER AND LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, p. 64–94.
3. KENNETH I. APPEL AND J. BARKLEY ROSSER, *Table for Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.*, v. 16, 1962, p. 500–501.
4. A. E. INGHAM, *The Distribution of Prime Numbers*, Cambridge Tract No. 30, Cambridge University Press, 1932.

40[F].—J. BARKLEY ROSSER & LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, Tables I-IV on p. 90–93.

The four number-theoretic tables reviewed here were presented by the authors in connection with their proofs of numerous inequalities concerning the distribution of primes. These inequalities include

$$\frac{x}{\log x}\left(1 + \frac{1}{2 \log x}\right) < \pi(x) < \frac{x}{\log x}\left(1 + \frac{3}{2 \log x}\right) \qquad (59 \leqq x),$$